CS311 Quiz#6 (winter 99)

Student Name (please print):

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Problem. Consider the grammar G:

$$
\begin{array}{lllll}
G: & \text{S} & \rightarrow & \text{AB} \mid \text{C} \\
 & \text{A} & \rightarrow & \text{aA} \mid \text{B} \\
 & \text{B} & \rightarrow & \text{bB} \mid \text{C} \\
 & \text{C} & \rightarrow & \text{cC} \mid \text{a} \mid \text{A}.
\end{array}
$$

Give a regular expression for $L(G)$ and construct an equivalent grammar G_C that does not contain chain rules.

$$
\begin{array}{lll}
\text{A} & \stackrel{*}{\Rightarrow} & (a^*b^*c^*)^+a = (a \cup b \cup c)^*a \\
\text{B} & \stackrel{*}{\Rightarrow} & b^*c^*A \stackrel{*}{\Rightarrow} b^*c^*(a \cup b \cup c)^*a \\
\text{C} & \stackrel{*}{\Rightarrow} & c^*A \stackrel{*}{\Rightarrow} c^*(a \cup b \cup c)^*a
\end{array}
$$

Therefore we have

$$
\begin{array}{lll}
\text{S} & \rightarrow & \text{C} \stackrel{*}{\Rightarrow} c^*(a^*b^*c^*)^+a = c^*(a \cup b \cup c)^*a \\
\text{S} & \rightarrow & \text{AB} \stackrel{*}{\Rightarrow} (a \cup b \cup c)^*ab^*c^*(a \cup b \cup c)^*a
\end{array}
$$

In consequence

$$L(G) = ((a \cup b \cup c)^*ab^* \cup \lambda)c^*(a \cup b \cup c)^*a$$

Using regular expression identities this can be reduced to

$$L(G) = (a \cup b \cup c)^*a$$

$$
\begin{array}{lllll}
G_C: & \text{S} & \rightarrow & \text{AB} \mid \text{aA} \mid \text{bB} \mid \text{cC} \mid \text{a} \\
 & \text{A} & \rightarrow & \text{aA} \mid \text{bB} \mid \text{cC} \mid \text{a} \\
 & \text{B} & \rightarrow & \text{aA} \mid \text{bB} \mid \text{cC} \mid \text{a} \\
 & \text{C} & \rightarrow & \text{aA} \mid \text{bB} \mid \text{cC} \mid \text{a}\ .
\end{array}
$$