CS311 Quiz#5 (winter 99)

Student Name (please print):

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Problem 1. Consider the grammar:

$$\begin{array}{lll} S & \rightarrow & aSB \mid aA \mid \lambda \\ A & \rightarrow & aA \mid aAb \mid bB \\ B & \rightarrow & bB \mid b. \end{array}$$

1. Prove that it is ambiguous.
2. Describe L(G) in set or regular experession notation.
3. Give an equivalent unambiguous grammar.

1. (3 pts) Ambiguity: w=abb has 2 leftmost derivations.

- $S \Rightarrow aSB \Rightarrow aB \Rightarrow abB \Rightarrow abb$ and
- $S \Rightarrow aA \Rightarrow abB \Rightarrow abb.$

2. (3 pts) Language. Starting with S there are several possible paths:

- $S \Rightarrow \lambda$. Therefore $\lambda \in L(G)$.
- $S \overset{n\geq 0}{\Rightarrow} a^nSB^n \Rightarrow a^nB^n \overset{*}{\Rightarrow} a^n(b^+)^n$. Therefore $\{a^ib^j | j \geq i \geq 1\} \subset L(G)$.
- $S \overset{n}{\Rightarrow} a^nSB^n \Rightarrow a^{n+1}AB^n \overset{m}{\Rightarrow} a^{n+m+1}AB^n \overset{p}{\Rightarrow} a^{n+m+p+1}Ab^pB^n \Rightarrow a^{n+m+p+1}bBb^pB^n \overset{q}{\Rightarrow} a^{n+m+p+1}bb^qBb^pB^n \Rightarrow a^{n+m+p+1}bb^qbb^pB^n \overset{*}{\Rightarrow} a^{n+m+p+1}b^{q+p+n+r+2}$. Therefore $\{a^ib^j | i \geq 1, j \geq 2\} \subset L(G)$.

Since all the strings $\{a^i b^j | i \geq 1, j \geq 2\} \subset L(G)$, the question is what happens for $j = 0, 1$. It is clear that $j = 0$ means a string of the form a^i, and λ is the unique string of this form in $L(G)$. By the second point above $\{a^i b^j | j \geq i \geq 1\} \subset L(G)$, therefore $j = 1$ requires $i = 1$ also, and $ab \in L(G)$. In consequence

$$L(G) \;=\; \lambda \,\cup\, ab \,\cup\, a^+ b b^+ \ .$$

3. (4 pts) A regular, unambiguous, equivalent grammar is:

$$\begin{array}{rcl} S & \rightarrow & \lambda | aC | aA \\ C & \rightarrow & b \\ A & \rightarrow & aA | bB \\ B & \rightarrow & bB | b \end{array}$$