CS311 Quiz#2 (winter 99)

Student Name (please print):

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Problem 1. Prove that, for any natural number $n \in \mathbf{N}$

$$1 + 3 + 9 + \ldots + 3^n = \frac{3^{n+1} - 1}{2} .$$

Hint: use mathematical induction. Take $n = 0$ as the anchor (basis) case.

Basis: n=0. 1 = (3-1)/2

Inductive Hypothesis: Suppose

$$1 + 3 + 9 + \ldots + 3^k = \frac{3^{k+1} - 1}{2} \text{ ,for all } k = 0, \ldots, n.$$

Induction Step: The $n + 1^{\text{st}}$ sum is

$$(1 + 3 + 9 + \ldots + 3^n) + 3^{n+1} = (\textit{by induction hypothesis})$$

$$= \frac{3^{n+1} - 1}{2} + 3^{n+1} = \frac{3^{n+1} - 1 + 2 \cdot 3^{n+1}}{2} = \frac{3 \cdot 3^{n+1} - 1}{2} = \frac{3^{n+2} - 1}{2}$$

Problem 2. Give a recursive definition of the set of all the descendants of a node x in a tree (the "subtree" generated by x). *Hint: use the [parent, child] relation as the operation in the definition. Start with* $X_0 = (\{x\}, \phi, x)$.

Denote the whole tree by $\mathcal{T} = (\mathcal{N}, \mathcal{A}, r)$.

Denote $X = (N, A, x)$ a subtree with root x, i.e. $N \subset \mathcal{N}$ and $A \subset \mathcal{A}$ and X itself is a tree (its root is x, all the elements of N are descendants of x and A represents the set of arcs between these nodes).

For the solutions you receive(d) full grade; for the complete solution you receive(d) full grade and a bonus point.

Simple Solution:

Let D be the set of descendants of x.

Basis: $x \in D$.

Recursive Step: If $a \in D$ and $[a, b] \in \mathcal{A}$ (i.e. a is a parent of b in T) then $b \in D$.

Closure: $c \in D$ only if it can be reached by a finite number of recursive steps.

Complete Solution:

Basis: $X_0 = (\{x\}, \phi, x)$ is a subtree (x is a descendant of itself).

Recursive Step: If $X = (N, A, x)$ is a subtree, $a \in N$ and $[a, b] \in \mathcal{A}$ (i.e. a is a parent of b in T), then $X' = (N \cup \{b\}, A \cup \{[a, b]\}, x)$ is a subtree. In particular, b is a descendant of x.

Closure: Any subtree can be obtained from X_0 by finitely many applications of the recursive step. In particular, any descendant can be obtained from x by finitely many applications of the recursive step.