CS 4414, Spring 2012
Homework #4

April 17, 2012

Consider the Runge function

$$f(x) = \frac{1}{1+x^2}, \quad x \in [a, b] \tag{1}$$

where $a = -5$ and $b = 5$.

Problem 1

Consider the interpolation points

$$x_1 = a, \quad x_2 = (a+b)/2, \quad x_3 = b.$$

Construct the quadratic interpolant for the Runge function at x_0, x_1, x_2 using:

1. the direct approach (Vandermonde matrix)

2. Lagrange interpolation

3. Newton divided differences interpolation

Problem 2

Write a program to compute the interplant of degree n at the points $x_1, \ldots, x_{n+1}$. Plot the function and the polynomial interpolant on the same graphic for $n = 4, 6, 10$.

1. Choose equidistant points

$$x_i = a + \frac{b-a}{n}(i-1), \quad i = 1, \ldots, n+1.$$

2. Choose the points to be the projections of equally distributed points along a half circle:

$$x_i = \frac{b-a}{2} \cos\left(\frac{\pi}{n}(i-1)\right), \quad i = 1, \ldots, n+1.$$

Comment on the differences you observe between the quality of the interpolation with the two choices of points.