How to Solve Hard Problems by Making Them Harder

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Research Areas

- Numerical analysis
- Nonlinear programming
- Parallel computing
- Mathematical software
- Image processing/computer vision
- Solid mechanics
- Fluid mechanics
- Computational biology/bioinformatics
- Multidisciplinary engineering design
- Scientific computing

Center Memberships

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Nonlinear programming is

1) unstructured computer programming using GO TOs;

2) a variant of extreme programming;

3) how computers were programmed in the 1960s;

4) the study of nonlinear constrained optimization problems.

Homotopy Algorithms

Let $x = (x_1, \ldots, x_n) \in E^n$, $F : E^n \to E^n$ be C^2.

Hard problem: $F(x) = 0$.

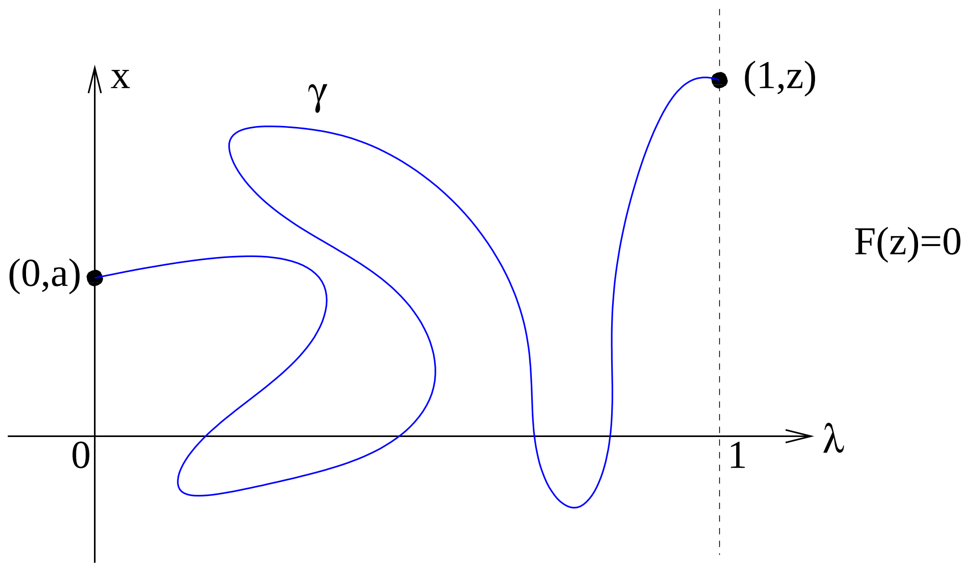
Newton's method: $x^{(k+1)} = x^{(k)} - \left(DF(x^{(k)})\right)^{-1} F(x^{(k)})$.

Homotopy method: let $S(x) = 0$ be a simple problem with a unique solution a (e.g., $S(x) = x - a = 0$).

Construct the homotopy map

$$\rho_a(\lambda, x) = \lambda F(x) + (1 - \lambda) S(x), \qquad 0 \le \lambda \le 1.$$

Track the zero curve γ of $\rho_a(\lambda, x) = 0$ from $\lambda = 0$ to $\lambda = 1$.



Under certain assumptions on F and S, this works with probability one (for almost all a).

Linear Complementarity Problem (LCP)

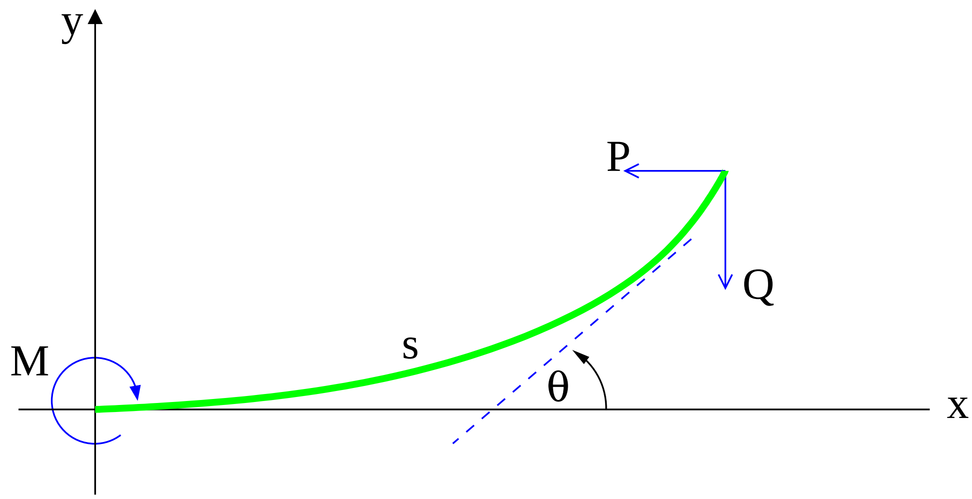
$$w - Mz = q, \qquad w \geqq 0, \qquad z \geqq 0, \qquad w^t z = 0.$$

$$Ax = q, \quad x \geqq 0, \quad A_{\cdot i} \in \{I_{\cdot i}, -M_{\cdot i}\}, \quad 2^n \text{ systems.}$$

$$K_i(z) = (Mz + q)_i^3 + z_i^3 - |(Mz + q)_i - z_i|^3 = 0$$
$$\iff w_i \geqq 0, \quad z_i \geqq 0, \quad w_i z_i = 0.$$

$$\rho_a(\lambda, z) = \lambda K(z) + (1 - \lambda)(z - a), \qquad 0 \leqq \lambda \leqq 1.$$

Elastic rod



$$\frac{d\theta}{ds} = Qx - Py + M, \qquad \frac{dx}{ds} = \cos\theta, \qquad \frac{dy}{ds} = \sin\theta.$$

$$x(0) = y(0) = \theta(0) = 0,$$

$$x(1) = a, \qquad y(1) = b, \qquad \theta(1) = c.$$

$$v = \begin{pmatrix} Q \\ P \\ M \end{pmatrix}, \qquad F(v) = \begin{pmatrix} x(1; v) - a \\ y(1; v) - b \\ \theta(1; v) - c \end{pmatrix} = 0.$$

$$\rho(\lambda, v) = \begin{pmatrix} x(1; v) - [\lambda a + (1-\lambda)1] \\ y(1; v) - [\lambda b + (1-\lambda)0] \\ \theta(1; v) - [\lambda c + (1-\lambda)0] \end{pmatrix} = 0, \qquad 0 \le \lambda \le 1.$$